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Flux-lines through Calabi–Yau manifolds and related couplings

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Abstract. The mixed Yukawa couplings of the 27 and 27* superfields to other superfields are studied in a model stemming from a superstring model compactified on a generic Calabi–Yau manifold. The coupling to the standard E_6-algebra-valued flux-loop operator, as well as an analogous E_6-invariant operator, is argued to be related to certain Yukawa couplings. A vacuum expectation value of the latter operator is shown to give masses to all E_6-singlet scalar superfields, except for those stemming from the supergravity multiplet.

1. Introduction

It is by now standard [1] that, in an effective model stemming from a $(9+1)$-dimensional spacetime heterotic superstring compa[illegible]alabi–Yau manifold ($\mathcal{M}$), the spectrum of *massless* superfields is obtained in terms of certain *harmonic* forms on $\mathcal{M}$. The fields which are phenomenologically most interesting are found as components of the $E_8 \times E_8$-algebra-valued connection 1-form‡, restricted to the complex three-dimensional internal $\mathcal{M}$. Using the symmetry which reduces to *CPT* in four dimensions, it suffices to consider only antiholomorphic 1-forms, and hence elements of the Dolbeault cohomology groups.

Initially, these forms are valued in the $E_8 \times E_8$ bundle, the fibres of which transform as the adjoint $(248, 1) \oplus (1, 248)$. To ensure the cancellation of the Yang–Mills and gravitational anomalies, one imposes a constraint on the space of background spin and Yang–Mills connections. This has the important consequence that the components of the fibres of the $E_8 \times E_8$ bundle, which transform as 3 of $SU(3) \subset E_8$, get identified with fibres of $\mathcal{T}_{\mathcal{M}}$, the tangent bundle of $\mathcal{M}$ (the holonomy of which is SU(3), from the definition of a Calabi–Yau manifold). We hereafter focus on antiholomorphic 1-forms on $\mathcal{M}$ valued in

$$\begin{aligned} \mathcal{V} &\to \operatorname{End} \mathcal{E} \oplus \mathcal{E} \otimes \mathcal{T}_{\mathcal{M}} \oplus \mathcal{E}^* \otimes \mathcal{T}^*_{\mathcal{M}} \oplus \operatorname{End} \mathcal{T}_{\mathcal{M}} \\ 248 &\to (78, 1) \oplus (27, 3) \oplus (27^*, 3^*) \oplus (1, 8) \end{aligned} \tag{1.1}$$

where 248 represents the adjoint of E_8; the 'other' E_8, that commutes with SU(3), is irrelevant in the analysis below and we ignore it hereafter. Note that 'End' here and in the following denotes the group of only traceless endomorphisms.

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‡ The analysis is straightforwardly extended to include fermionic superpartners, so we focus here on the bosonic modes only. Recasting the results in a manifestly supersymmetric form is then easily achieved.

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Upon a harmonic analysis on $\mathcal{M}$, the relevant components of the Yang–Mills connection 1-form in $(9+1)$-dimensional spacetime, subject to the restrictions described above, are

$$\begin{aligned} &A^{I\bar{I}}_{\bar{\mu}}(x;y) = && \text{massive} && \\ &A^{Ia}_{\bar{\mu}}(x;y) = \Phi^I_i(x)u^{i,a}_{\bar{\mu}}(y) && +\text{massive} && i=1,\ldots,\dim H^1(\mathcal{M},\mathcal{T}_{\mathcal{M}}) \\ &A^{\bar{I}\bar{a}}_{\bar{\mu}}(x;y) = \bar{\Phi}^{\bar{I}}_{\bar{i}}(x)v^{\bar{i},\bar{a}}_{\bar{\mu}}(y) && +\text{massive} && \bar{i}=1,\ldots,\dim H^1(\mathcal{M},\mathcal{T}^*_{\mathcal{M}}) \\ &A^{a\bar{a}}_{\bar{\mu}}(x;y) = \tilde{\Phi}_{\tilde{n}}(x)\varphi^{\tilde{n},a\bar{a}}_{\bar{\mu}}(y) && +\text{massive} && \tilde{n}=1,\ldots,\dim H^1(\mathcal{M},\mathrm{End}\,\mathcal{T}_{\mathcal{M}}) \end{aligned} \tag{1.2}$$

where we have used greek, lower case and capital latin letters to denote covariant (form), contravariant (tangent) indices on $\mathcal{M}$ and E_6 indices, respectively, while $(x;y)$ denote the real four- and six-dimensional coordinates on the (local) product of the Minkowski and Calabi–Yau space. The forms u, v and φ are harmonic with respect to the $\bar{\partial}$ operator on $\mathcal{M}$, the antiholomorphic part of the real exterior derivative, and are therefore elements of the Dolbeault cohomology groups indicated on the right of equations (1.2).

Cubic Yukawa couplings of $\Phi^I_i(x)$ and those of $\bar{\Phi}^{\bar{I}}_{\bar{i}}(x)$ are shown to arise from the original tree-level action [2] and the corresponding coupling coefficients were shown to be given by integrals on $\mathcal{M}$, over triple products of $u^i(y)$ and $v^{\bar{i}}(y)$ respectively. Our aim here is to derive mixed couplings of $\Phi^I_i(x)$ and $\bar{\Phi}^{\bar{I}}_{\bar{i}}(x)$ and relate their coupling coefficients to certain integrals over various forms on $\mathcal{M}$. In §§ 2, 3 and 4 we do this for mixed couplings to other massless modes on $\mathcal{M}$, and then also for the case of massive modes. The latter we show to reflect the effect of the so-called flux-loops. In § 5 we discuss the phenomenological impact of the couplings derived and comment on certain analogies with recent exact string-theory results. Our concluding remarks constitute § 6 and some technical details are left for the appendix.

In what follows we shall use only *general* features of Calabi–Yau manifolds. In particular, there exists a covariantly constant harmonic (3, 0)-form, represented by its tensor coefficient $\omega_{\mu\nu\sigma}$, its complex conjugate $\bar{\omega}_{\bar{\mu}\bar{\nu}\bar{\sigma}}$ and the dual rank-3 totally anti-symmetric tensors ε^{abc} and $\bar{\varepsilon}^{\bar{a}\bar{b}\bar{c}}$. Moreover, employing the orthonormality relation given by the Hermitian inner product

$$\delta^{\mu}{}_{a} := \langle \mathrm{d}z^{\mu}, \partial_a \rangle$$

one may define its 'inverse':

$$\delta_{\mu}{}^{a} := \omega_{\mu\nu\sigma}\delta^{\nu}{}_{b}\delta^{\sigma}{}_{c}\varepsilon^{abc}$$

together with the appropriate conjugates. We lower and raise tangent indices using the flat tangent metric $\eta_{a\bar{a}}$ and its inverse $\eta^{a\bar{a}}$. The isomorphisms

$$H^1(\mathcal{M},\mathcal{T}_{\mathcal{M}}) \approx H^{2,1}(\mathcal{M})$$

$$H^1(\mathcal{M},\mathcal{T}^*_{\mathcal{M}}) = H^{1,1}(\mathcal{M}) \stackrel{*}{\approx} H^{2,2}(\mathcal{M}) \approx H^2(\mathcal{M},\mathcal{T}_{\mathcal{M}}) \tag{1.3}$$

shall be used as well.

2. Mixed couplings

The massless four-dimensional fields correspond to 0-modes of the Fourier expansion on $\mathcal{M}$ and therefore to harmonic forms as in equation (1.2). The groups $H^1(\mathcal{M},\mathcal{T}_{\mathcal{M}})$,

$H^1(\mathcal{M}, \mathcal{T}^*_{\mathcal{M}})$ and $H^1(\mathcal{M}, \mathrm{End}\, \mathcal{T}_{\mathcal{M}})$ are of course 0-eigenspaces of the Laplace operator on $\mathcal{M}$, but they are not irreducible. In fact all three of them are reducible [3] and naturally decompose into irreducible representations of the Holonomy group, $SU(3)_H$. This decomposition is straightforwardly obtained by using the

$$\begin{aligned} \mathcal{P}_{6\bar{\mu}c}^{\bar{\pi}a} &:= [\tfrac{1}{2}(\delta^{\bar{\pi}}_{\bar{\mu}}\delta^{a}_{c} + \delta^{\bar{\pi}a}\delta_{\bar{\mu}c})] \\ \mathcal{P}_{3^*\bar{\mu}c}^{\bar{\pi}a} &:= [\tfrac{1}{2}(\delta^{\bar{\pi}}_{\bar{\mu}}\delta^{a}_{c} - \delta^{\bar{\pi}a}\delta_{\bar{\mu}c})] \end{aligned} \tag{2.1}$$

projectors on $u^{i,c}_{\bar{\pi}}\, d\bar{z}^{\bar{\pi}} \in H^1(\mathcal{M}, \mathcal{T}_{\mathcal{M}})$, the

$$\begin{aligned} \mathcal{P}_{8\bar{\sigma}\bar{c}}^{\bar{\tau}\bar{a}} &:= [(\delta^{\bar{\tau}}_{\bar{\sigma}}\delta^{\bar{a}}_{\bar{c}} - \tfrac{1}{3}\delta_{\bar{\sigma}}{}^{\bar{a}}\delta^{\bar{\tau}}{}_{\bar{c}})] \\ \mathcal{P}_{\bar{\sigma}\bar{c}}^{\bar{\tau}\bar{a}} &:= [(\tfrac{1}{3}\delta_{\bar{\sigma}}{}^{\bar{a}}\delta^{\bar{\tau}}{}_{\bar{c}})] \end{aligned} \tag{2.2}$$

projectors on $v^{\bar{i},\bar{c}}_{\bar{\tau}}\, d\bar{z}^{\bar{\tau}} \in H^1(\mathcal{M}, \mathcal{T}^*_{\mathcal{M}})$, and the

$$\begin{aligned} \mathcal{P}_{15\bar{\nu}d\bar{d}}^{\bar{\rho}b\bar{b}} &:= [\tfrac{1}{2}(\delta^{\bar{\rho}}_{\bar{\nu}}\delta^{b}_{d} + \delta^{\bar{\rho}b}\delta_{\bar{\nu}d})\delta^{\bar{b}}_{\bar{d}} - \tfrac{1}{8}(\delta_{\bar{\nu}}{}^{\bar{b}}\delta^{b}_{d} + \eta^{b\bar{b}}\delta_{\bar{\nu}d})\delta^{\bar{\rho}}{}_{\bar{d}}] \\ \mathcal{P}_{6^*\nu d\bar{d}}^{\bar{\rho}bb} &:= [\tfrac{1}{2}(\delta^{\bar{\rho}}_{\bar{\nu}}\delta^{b}_{d} - \delta^{\rho b}\delta_{\nu d})\delta^{\bar{b}}_{\bar{d}} - \tfrac{1}{4}(\delta_{\bar{\nu}}{}^{\bar{b}}\delta^{b}_{d} - \eta^{b\bar{b}}\delta_{\bar{\nu}d})\delta^{\bar{\rho}}{}_{\bar{d}}] \\ \mathcal{P}_{3\nu d\bar{d}}^{\rho b\bar{b}} &:= [\tfrac{1}{8}(3\delta_{\nu}{}^{\bar{b}}\delta^{b}_{d} - \eta^{b\bar{b}}\delta_{\bar{\nu}d})\delta^{\bar{\rho}}_{\bar{d}}] \end{aligned} \tag{2.3}$$

projectors on $A^{\bar{n}.d\bar{d}}_{\bar{\rho}}(x;y)$. Note that it follows from the Lefschetz decomposition of forms that

$$\begin{aligned} &\mathcal{P}_3 H^1(\mathcal{M}, \mathrm{End}\,\mathcal{T}_{\mathcal{M}}) \overset{*}{\approx} \mathcal{P}_{3^*}H^1(\mathcal{M}, \mathcal{T}_{\mathcal{M}}) \overset{*}{\approx} \mathcal{P}_3 H^1(\mathcal{M}) = H^1(\mathcal{M}) = \varnothing \\ &\mathcal{P}_{6^*}H^1(\mathcal{M}, \mathrm{End}\,\mathcal{T}_{\mathcal{M}}) \overset{*}{\approx} \mathcal{P}_{3^*}H^1(\mathcal{M}, \mathcal{T}_{\mathcal{M}}) = H^1(\mathcal{M}, \mathcal{T}_{\mathcal{M}}) \\ &\mathcal{P}_1 H^1(\mathcal{M}, \mathcal{T}^*_{\mathcal{M}}) \approx \mathcal{P}_1 H^0(\mathcal{M}) = H^0(\mathcal{M}). \end{aligned} \tag{2.4}$$

It is then straightforward to identify, without loss of generality, the Kähler (1, 1)-form with $v^1 \in \mathcal{P}_1 H^1(\mathcal{M}, \mathcal{T}^*_{\mathcal{M}})$, while $v^{\bar{i}} \in \mathcal{P}_8 H^1(\mathcal{M}, \mathcal{T}^*_{\mathcal{M}})$ for $\bar{i} = \hat{i} = 2, \ldots, b_{1.1}$.

Following [2], we start with the interaction term in the original tree-level action:

$$\mathcal{A}_{\rm int} = \int d^{10}X\, \sqrt{-g_{10}}\, \mathrm{Tr}\{\overline{\lambda(X)} \cdot \not{A}(X) \cdot \lambda(X)\}.$$

Here we project $\Phi^I_i(x) \cdot u^i|_6(y)$ from $\lambda(X)$, $\bar{\Phi}^{\bar{I}}_{\bar{i}}(x) \cdot v^{\bar{i}}|_{1\oplus 8}(y)$ from $\overline{\lambda(X)}$ and keep only the integration on $\mathcal{M}$. This yields the following generic coupling:

$$\begin{aligned} \Phi^I_i(x)\bar{\Phi}^{\bar{I}}_{\bar{i}}(x) \cdot &\int_{\mathcal{M}} \bar{\varepsilon}^{\bar{\mu}\bar{\nu}\bar{\sigma}} u^{i.a}_{\bar{\mu}}(y)\eta_{a\bar{b}} A_{\bar{\nu}}{}^{b\bar{b}}_{I\bar{I}}(x;y)\eta_{b\bar{a}} v^{\bar{i}.\bar{a}}_{\bar{\sigma}} \\ \cdot &\int_{\mathcal{M}} \bar{\varepsilon}^{\bar{\mu}\bar{\nu}\bar{\sigma}} u^{i}_{(\bar{\mu}}{}^{a}(y)\delta_{\bar{\pi})a}\delta^{\bar{\pi}}{}_{\bar{b}} A_{\bar{\nu}}{}^{b\bar{b}}_{I\bar{I}}(x;y)\eta_{b\bar{a}} v^{\bar{i}.\bar{a}}_{\bar{\sigma}} \end{aligned} \tag{2.5}$$

where the projection to the harmonic (6-) subset of the $\mathcal{T}_{\mathcal{M}}$-valued (0, 1)-forms is made explicit in the second line, $A_{\bar{\nu}}{}^{b\bar{b}}_{I\bar{I}}(x;y)$ includes the first and the last set of fields in (1.2), $\eta^{b\bar{b}} A_{\bar{\nu}I\bar{I}}(x;y) \oplus \eta_{I\bar{I}} A_{\bar{\nu}}{}^{b\bar{b}}(x;y)$ and we have abbreviated

$$\bar{\varepsilon}^{\bar{\mu}\bar{\nu}\bar{\sigma}} := \delta^{\bar{\mu}}{}_{\bar{a}}\delta^{\bar{\nu}}{}_{\bar{b}}\delta^{\bar{\sigma}}{}_{\bar{c}}\bar{\varepsilon}^{\bar{a}\bar{b}\bar{c}}.$$

As massless modes of $A_{\bar{\nu}I\bar{I}}(x;y)$ would correspond to elements of $H^1(\mathcal{M})$, which is empty, $A_{\bar{\nu}I\bar{I}}(x;y)$ expands into massive modes only. We shall return to these later and now we comment on the $A_{\bar{\nu}}{}^{b\bar{b}}(x;y)$.

Since $A_{\bar\nu}{}^{b\bar b}(x; y) \cdot \eta_{b\bar b} = 0$, the 0-modes correspond to $\mathrm{H}^1(\mathcal{M}, \mathrm{End}\, \mathcal{T}_{\mathcal{M}})$ for which there is no straightforward method in general to obtain the rank [4]. Employing the equivalences in (2.4) a lower bound is, however, obtained [3]:

$$\mathrm{rank}\, \mathrm{H}^1(\mathcal{M}, \mathrm{End}\, \mathcal{T}_{\mathcal{M}}) \geqslant \mathrm{rank}\, \mathrm{H}^1(\mathcal{M}, \mathcal{T}_{\mathcal{M}}) \tag{2.6}$$

because $\mathrm{rank}\, \mathcal{P}_{15}\mathrm{H}^1(\mathcal{M}, \mathrm{End}\, \mathcal{T}_{\mathcal{M}}) \geqslant 0$. Interestingly enough this result is confirmed away from the point-field theory limit, in which (2.6) was first derived, by considering fully fledged string theory on orbifolds which are resolved into Calabi–Yau manifolds and the analysis is valid through all finite orders of a perturbation series in the sizes of the resolutions [5]. The fact that *exact* string-theory results on a restricted class of Calabi–Yau manifolds and point-field-theoretic results for generic Calabi–Yau manifolds coincide we find, while certainly not a full proof, at least a strong motivation for the subsequent analysis.

3. Massless fields coupling to '27 · 27*'

The 0-modes of $A_{\bar\nu}{}^{b\bar b}(x; y)$ are found to correspond to elements of $\mathcal{P}_{15}A_{\bar\nu}{}^{b\bar b}(x; y)$ and of $\mathcal{P}_{6^*}A_{\bar\nu}{}^{b\bar b}(x; y)$, the latter of which is, by equation (2.4), equivalent to the conjugates of elements of $\mathrm{H}^1(\mathcal{M}, \mathcal{T}_{\mathcal{M}})$ which, in turn, are used to describe the '27'. While we shall have nothing more concrete to say here regarding the modes of $\mathcal{P}_{15}A_{\bar\nu}{}^{b\bar b}(x; y)$, it follows that the $\mathcal{P}_{6^*}A_{\bar\nu}{}^{b\bar b}(x; y)$ may be treated in full analogy with the '27'. It has, on the other hand, recently been proved [6] that elements of $\mathrm{H}^1(\mathcal{M}, \mathcal{T}_{\mathcal{M}})$ may be parametrised, for a very large class of Calabi–Yau manifolds, by certain polynomials [7]. This parametrisation enables one, in principle, to study the action of any symmetry of $\mathcal{M}$ on $\mathrm{H}^1(\mathcal{M}, \mathcal{T}_{\mathcal{M}})$ and therefore, via the second relation in (2.4), on $\mathcal{P}_{6^*}\mathrm{H}^1(\mathcal{M}, \mathrm{End}\, \mathcal{T}_{\mathcal{M}})$. More concretely, for $\mathcal{D}$ a (discrete) symmetry of $\mathcal{M}$:

$$\left.\begin{array}{l} \mathrm{H}^1(\mathcal{M}, \mathcal{T}_{\mathcal{M}}) \ni u^i \sim \boldsymbol{R}_i \\ \Rightarrow \mathcal{P}_{6^*}\mathrm{H}^1(\mathcal{M}, \mathrm{End}\, \mathcal{T}_{\mathcal{M}}) \ni \varphi^i \sim \boldsymbol{R}_i^* \end{array}\right\} \text{under } \mathcal{D}.$$

As the transformation properties of $v^{\bar i} \in \mathrm{H}^1(\mathcal{M}, \mathcal{T}^*_{\mathcal{M}})$ may be determined by indirect methods [8], the discrete symmetries may be used to restrict not only the '27^3' and '27^{*3}' couplings but also the $(u^i\varphi^i v^{\bar i}) \subset$ '27 · 1 · 27*' mixed couplings. The remaining couplings to $\varphi' \in \mathcal{P}_{15}A_{\bar\nu}{}^{b\bar b}(x; y)$ are of course restricted as well, but there does not seem to exist a systematic and universal method to derive the transformation properties of these fields under discrete symmetries of $\mathcal{M}$.

Both $(u^i\varphi^j v^{\bar i})$ and $(u^i\varphi'^{j'} v^{\bar i})$ follow from equation (2.5) and we include their explicit form for completeness:

$$(u^i\varphi^j v^{\bar i}) = \tfrac{1}{4}\int_{\mathcal{M}} (\bar\varepsilon^{\bar\mu\bar\nu\bar\sigma}\delta_{\bar a}^{\bar c}\eta_{a\bar b} - \bar\varepsilon^{\bar\mu}{}_a{}^{\bar\sigma}\delta_{\bar a}^{\bar c}\delta^{\bar\nu}{}_{\bar b} + \tfrac{1}{2}\bar\varepsilon^{\bar\mu}{}_a{}^{\bar\sigma}\delta_{\bar b}^{\bar c}\delta^{\bar\nu}{}_{\bar a})(\eta_{c(\bar c}u^{i.}_{\bar\mu)}{}^{c})\varphi^{j.a\bar a}_{\bar\nu} v^{\hat i.\bar b}_{\bar\sigma}$$

$$= \tfrac{1}{6}\int_{\mathcal{M}} \bar\varepsilon^{\bar\mu\bar\nu\bar\sigma}(\eta_{a(\bar c}u^{i.}_{\bar\mu)}{}^{a})(\varphi^{j,c\bar c}_{\nu}\delta_{\bar\sigma c})v^1 \qquad \hat i = 2, \ldots, \mathrm{rank}\, \mathrm{H}^1(\mathcal{M}, \mathcal{T}^*_{\mathcal{M}})$$

where $\hat i$ labels elements of $\mathcal{P}_8\mathrm{H}^1(\mathcal{M}, \mathcal{T}^*_{\mathcal{M}})$. We also obtain

$$(u^i\varphi'^{j'} v^{\bar i}) = \tfrac{1}{4}\int_{\mathcal{M}} [\bar\varepsilon^{\bar\mu\bar\nu\bar\sigma}\delta_{\bar a}^{\bar c}\eta_{a\bar b} + \bar\varepsilon^{\bar\mu}{}_a{}^{\bar\sigma}\delta_{\bar a}^{\bar c}\delta^{\bar\nu}{}_{\bar b} - \tfrac{1}{4}\bar\varepsilon^{\bar\mu}{}_a{}^{\bar\sigma}\delta_{\bar b}^{\bar c}\delta^{\bar\nu}{}_{\bar a}](\eta_{c(\bar c}u^{i.}_{\bar\mu)}{}^{c})\varphi^{\bar j.a\bar a}_{\bar\nu} v^{\hat i.\bar b}_{\bar\sigma}$$

$$= 0 \qquad \text{for } v = v^1$$

where the second line follows essentially from $6 \otimes 15 \not\ni 1$ in SU(3).

The explicit expressions for the coupling constants appear to be quite cumbersome and not very helpful unless a parametrisation is provided so that the integrals can actually be evaluated. It must be borne in mind that these expressions are just projections of the quite simple looking integral

$$(u^i \varphi^{\tilde{n}} n^{\bar{i}}) = \int_{\mathcal{M}} \bar{\varepsilon}^{\bar{\mu}\bar{\nu}\bar{\sigma}} u_{\bar{\mu}}^{i.a} \eta_{a\bar{b}} \varphi_{\bar{\nu}}^{\tilde{n}.b\bar{b}} \eta_{b\bar{b}} v_{\bar{\sigma}}^{\bar{i}.\bar{a}} \qquad \tilde{n} = j \oplus j'$$

which is the explicit representation of the map

$$\mathrm{H}^1(\mathcal{M}, \mathcal{T}_{\mathcal{M}}) \otimes \mathrm{H}^1(\mathcal{M}, \mathrm{End}\, T_{\mathcal{M}}) \otimes \mathrm{H}^1(\mathcal{M}, \mathcal{T}^*_{\mathcal{M}}) \to \mathbf{C}$$

as induced by the definition of End $\mathcal{T}_{\mathcal{M}}$, as well as the existence of a covariantly constant harmonic (3, 0)-form on any $\mathcal{M}$. With this in mind, the value of the above integrals depends only on the complex structure†.

The couplings of massless 27 and 27* to the 6*- and 15-sector of E_6-singlet matter superfields (those stemming from the Yang-Mills multiplet), with coupling coefficients $(u^i \varphi^j v^{\bar{i}})$ and $(u^i \varphi^{\prime ij} v^{\bar{i}})$, might indeed be phenomenologically relevant as they can provide the right-handed neutrinos with a heavy mass [10]. Since there are at least as many E_6-singlet superfields as there are 27 (see equation (2.6)), the mass matrix for the neutrinos, their mirrors and the E_6 singlets may indeed be realistic. Also, the '27 · 1 · 27*' terms in the superpotential, which correspond to the Yukawa couplings

$$\Phi_i^I(x) \tilde{\Phi}_{\tilde{n}}(x) \bar{\Phi}_{\bar{i}}^{\bar{I}}(x) \cdot \eta_{I\bar{I}} \cdot (u^i \varphi^{\tilde{n}} v^{\bar{i}})$$

induce

$$\left(\sum_{\tilde{n}} (u^i \varphi^{\tilde{n}} v^{\bar{i}})^\dagger (u^j \varphi^{\tilde{n}} v^{\bar{j}}) \right) [\Phi_i^I(x) \bar{\Phi}_{\bar{i}}^{\bar{I}}(x) \eta_{I\bar{I}}]^\dagger [\Phi_j^J(x) \bar{\Phi}_{\bar{j}}^{\bar{J}}(x) \eta_{J\bar{J}}] \qquad (3.1)$$

terms in the potential. Such $|27 \cdot 27^*|^2$ terms obviously play an important role in generating an intermediate scale.

The E_6 singlets, however, receive masses through two-dimensional world-sheet instanton effects which are proportional to the compactification mass scale but damped by a factor proportional to $\exp(-R/R_{Pl})$ with R being the size of the internal Calabi-Yau manifold [5]. This complicates the mass matrix and more detailed analysis in a concrete model is necessary, but it is possible to envision a pairing off of the E_6 singlets with the right-handed neutrinos, leaving (nearly) massless left-handed neutrinos. At the same time, since exponentially suppressed masses are of the order of the desired 'intermediate' mass scale in some models [11], the effect of the terms (3.1) is not negligible.

4. Massive fields coupling to '27 · 27*'

The massive modes of the various fields of course decouple from the low-energy particle spectrum but their possible vacuum expectation values (VEV) may have an important

† That this large degree of invariance is not spoiled by the projections (2.1)-(2.3), induced by the Lefschetz decomposition, follows from the highly topological properties of this decomposition itself; see, e.g., [9].

impact on the low-energy phenomenology. We now look for such effects by concentrating on the remaining two couplings which can be derived from (2.5). In particular, inserting $\eta^{b\bar{b}}A_{\bar{\nu}I\bar{I}}(x;y)$ for $A_{\bar{\nu}}{}^{b\bar{b}}_{I\bar{I}}(x;y)$ in (2.5) we obtain

$$\Phi^I_i(x)\bar{\Phi}^{\bar{I}}_{\bar{i}}(x)\int_{\mathcal{M}}\bar{\varepsilon}^{\bar{\mu}\bar{\nu}\bar{\sigma}}u^{i,a}_{\bar{\mu}}(y)\eta_{a\bar{a}}v^{\bar{i},\bar{a}}_{\bar{\sigma}}(y)\cdot A_{\bar{\nu}I\bar{I}}(x;y).$$

Let us now define

$$u^i_{\mu\nu\bar{\mu}}(y):=\omega_{\mu\nu\sigma}\delta^{\sigma}{}_a u^{i,a}_{\bar{\mu}}(y) \qquad u^{i,a}_{\bar{\mu}}(y)=u^i_{\mu\nu\bar{\mu}}(y)\delta^{\mu}{}_b\delta^{\nu}{}_c\varepsilon^{abc}$$
$$v^{\bar{i}}_{\sigma\bar{\sigma}}(y):=\delta_{\sigma\bar{a}}v^{\bar{i},\bar{a}}_{\bar{\sigma}}(y) \qquad v^{\bar{i},\bar{a}}_{\bar{\sigma}}(y)=v^{\bar{i}}_{\sigma\bar{\sigma}}(y)\delta^{\sigma\bar{a}}$$

which follow from the isomorphisms (1.3) and may be viewed as alternative definitions of $\delta^{\mu}{}_a$, $\delta_{\bar{\mu}}{}^{\bar{a}}$ and thus their conjugates as well. With these definitions, the above integral is

$$\int_{\mathcal{M}}\varepsilon^{\mu\nu\sigma}\bar{\varepsilon}^{\bar{\mu}\bar{\nu}\bar{\sigma}}u^i_{\mu\nu\bar{\mu}}(y)v^{\bar{i}}_{\sigma\bar{\sigma}}(y)\cdot A_{\bar{\nu}I\bar{I}}(x;y) \tag{4.1}$$

which we rewrite more symbolically as

$$\int_{\mathcal{M}}(2,1)^i\wedge(1,1)^{\bar{i}}\wedge A_{I\bar{I}}(x)=\int_{\mathcal{M}}(3,2)^{i\bar{i}}\wedge A_{I\bar{I}}(x)$$

with $A_{I\bar{I}}(x)=A_{\bar{\nu}I\bar{I}}(x;y)\,\mathrm{d}\bar{z}^{\bar{\nu}}$ being an E_6-algebra-valued (0, 1)-form on $\mathcal{M}$.

As indicated in the second expression, the wedge product $u^i\wedge v^{\bar{i}}$ defines a $b_{2,1}\times b_{1,1}$ matrix of (3, 2)-forms which are $\bar{\partial}$-closed since u and v are. The integral on $\mathcal{M}$ over a kernel defined by this matrix of (3, 2)-forms defines, together with the integrals over forms of conjugate type (see the appendix for a derivation), a set of integrals

$$\int_{\mathcal{M}}(3,2)^{i,\bar{i}}\wedge A_{I\bar{I}}(x:y)+\mathrm{CT}\simeq\oint_{\Gamma(i,\bar{i})}\mathrm{d}y^m\,A_{mI\bar{I}}(x;y) \qquad m=1,\ldots,6 \tag{4.2}$$

where y is the real six-dimensional coordinate on $\mathcal{M}$, and the closed contours $\Gamma(i,\bar{i})$ are defined by the matrix of (3, 2)-forms in the kernel of the integral (4.1)†.

The expression (4.1) is, in view of (4.2), formally the same as the exponent of the flux-loop operator

$$W(\Gamma):=P\exp\left(\mathrm{i}\oint_{\Gamma}\mathrm{d}y^m\,A^{(78)}_m(x;y)\right)=:P\exp(\mathrm{i}\mathscr{H}^{(78)}_{\Gamma}(x))$$

which is usually identified as the ordering parameter of the E_6 breaking [1, 12]. Here P is the path-ordering operator and Γ must be a non-contractible loop 'through' $\mathcal{M}$ if $W(\Gamma)\neq 1$. The contour integral over $A^{(78)}_m(x;y)$ may be viewed as an effective adjoint Higgs scalar in four dimensions. While the coupling of this effective Higgs scalar to the four-dimensional massless '27 · 27*' pairs is determined by the integral (4.1)‡, the VEV depends on the background configuration of $A^{(78)}_m(x;y)$ as much as on the contour(s).

† One may think of the contour integral as a generalisation of a 'surface term'.

‡ Viewed as a cubic Yukawa coupling in the superpotential, or the coupling of the 'effective' Higgs scalar to the superpartners of Φ and $\bar{\Phi}$, the VEV of the 'effective' Higgs scalar represents the mass of the '27 · 27*' pairs and the sign is irrelevant.

In complete analogy with the couplings to $A_{\bar{\nu}I\bar{I}}(x;y)$ we now derive the couplings to $\mathscr{P}_3 A_{\bar{\nu}}{}^{b\bar{b}}(x;y)$. Using the explicit form of $\mathscr{P}_3$ (2.3) we obtain

$$\Phi_i^I(x)\eta_{I\bar{I}}\bar{\Phi}_{\bar{i}}^{\bar{I}}(x)\int_{\mathscr{M}} \bar{\varepsilon}^{\bar{\mu}\bar{\nu}\bar{\sigma}} u_{\bar{\mu}}^{i,a}(y)\eta_{a\bar{a}} v_{\bar{\sigma}}^{\bar{i},\bar{a}}(y)\cdot A_{\bar{\nu}}^{(1)}(x;y)$$

where

$$A_{\bar{\nu}}^{(1)}(x;y) := -\tfrac{1}{16}\delta_{\bar{\nu}b}\delta^{\bar{\rho}}{}_{\bar{b}}A_{\bar{\rho}}{}^{b\bar{b}}(x;y)$$

results from the projections. Again, as in the case of $A_{\bar{\nu}I\bar{I}}(x;y)$, we rewrite the integral as

$$\int_{\mathscr{M}} \varepsilon^{\mu\nu\sigma}\bar{\varepsilon}^{\bar{\mu}\bar{\nu}\bar{\sigma}} u^{i}_{\mu\nu\bar{\mu}}(y) v^{\bar{i}}_{\sigma\bar{\sigma}}(y)\cdot A_{\bar{\nu}}^{(1)}(x;y) \tag{4.3}$$

which is completely analogous to (4.1) and leads to the *same* contour integral(s) as in equation (4.2), except that the integrand, and therefore the integral as well, is E_6 invariant.

We have thus derived the couplings of the '27 · 27*' pairs of massless fields to two flux-loop integrals:

$$\Phi_i^I(x)\bar{\Phi}_{\bar{i}}^{\bar{I}}(x)\cdot\left(\oint_{\Gamma(i,\bar{i})} dy^m\, A^{(78)}_{mI\bar{I}}(x;y)\oplus\eta_{I\bar{I}}\oint_{\Gamma(i,\bar{i})} dy^m\, A_m^{(1)}(x;y)\right) \tag{4.4}$$

starting from the tree-level action of the original point-limit of the heterotic superstring model in $(9+1)$-dimensional spacetime. *Assuming* suitable background configurations for $A^{(78)}$ and $A^{(1)}$, to provide the above flux-loop integrals with non-vanishing VEV in the $(3+1)$-dimensional spacetime sense, the phenomenological properties of a concrete effective model can be changed significantly.

5. Interpretation and phenomenological impact

Note that $\langle\mathscr{H}_\Gamma^{(78)}(x)\rangle := \langle\oint_\Gamma dy^m\, A_m^{(78)}(x;y)\rangle \neq 0$ is interpreted as the E_6-breaking ordering parameter, i.e. effective Higgs field in phenomenological applications which typically rely on point-limit analysis. It is crucial in this context as it is the only source of phenomenologically applicable breaking of E_6. Actually, $\langle\mathscr{H}_\Gamma^{(78)}(x)\rangle \neq 0$ cannot be a point-limit effect since it is defined to depend on a non-contractible path Γ. Since $\mathscr{H}_\Gamma^{(78)}(x)$ and the expression (4.1), in view of (4.2), are formally identical, we are tempted to identify them, i.e. to think of the flux-loop operator $W(\Gamma)$ as being generated by the expression (4.1) which we derived from the term in the point-limit action (2.5). Moreover, the expression (4.3) then defines a coupling of '27 · 27*' pairs to an analogous effective Higgs field, $\mathscr{H}_\Gamma^{(1)}(x)$.

Unlike $A_m^{(78)}$, $A_m^{(1)}$ makes no distinction between different components in the '27' and in the '27*', thus 'switching on' the singlet VEV seems to give superheavy mass to all '27 · 27*' pairs. This is however not quite so, and several remarks are in order.

(i) As long as $\Gamma(i\bar{i})$ are non-contractible, the contour integrals in (4.4) cannot be related by Stokes' theorem to any surface integral over the corresponding field strengths. Consequently, the value of $\langle\mathscr{H}_\Gamma^{(78)}\rangle$ and $\langle\mathscr{H}_\Gamma^{(1)}\rangle$ on one hand and those of $\langle F_{mn}^{(78)}\rangle$ and $\langle F_{mn}^{(1)}\rangle$ on the other can be chosen independently so as to be consistent with the standard ansatz. Finding explicit solutions for a background configuration of $\langle A_{\bar{\nu}}^{(78)}(x;y)\rangle$ and $\langle A_{\bar{\nu}}^{(1)}(x;y)\rangle$ such as to yield the desired pattern of VEV is certainly addressable only for concrete models.

(ii) A glance at the 'holomorphic part' of the contour integral in equations (4.2) and (4.1):

$$\oint_{\Gamma(i\bar{i})} \mathrm{d}\bar{z}^{\bar{\nu}} \stackrel{*}{\approx} \int_{\mathcal{M}} \varepsilon^{\mu\nu\sigma}\bar{\varepsilon}^{\bar{\mu}\bar{\nu}\bar{\sigma}} u^{i}_{\mu\nu\bar{\mu}} v^{\bar{i}}_{\sigma\bar{\sigma}} = \int_{\mathcal{M}} \bar{\varepsilon}^{\bar{\mu}\bar{\nu}\bar{\sigma}} u^{i.a}_{\bar{\mu}} \eta_{a\bar{a}} v^{\bar{i}.\bar{a}}_{\bar{\sigma}}$$

especially in its third form given here, reveals that, in the $\bar{i}=1$ case, when $v^{1.\bar{a}}_{\bar{\sigma}} \propto \delta_{\bar{\sigma}}{}^{\bar{a}}$ corresponding to the Kähler (1, 1)-form, the integral is projected onto $[(\mathcal{P}_{3*}u_{\bar{\mu}}{}^{a})\ldots]$ which have no 0-modes on any Calabi–Yau manifold. Therefore, the couplings we have derived from the original tree-level action do not couple the massless '27*' represented by the Kähler (1, 1)-form to massless '27' all of which are represented by $\mathcal{P}_6 u_{\bar{\mu}}{}^{a}$. If the effect of the standard (adj(E_6)-valued) flux-loop is indeed generated from this coupling, it leaves an entire '27*' massless!

(iii) Just as to '27 · 27*' pairs, $\mathcal{P}_3 A_{\bar{\nu}}{}^{a\bar{a}}(x;y)$ couples to $\mathcal{P}_{6*}A_{\bar{\nu}}{}^{a\bar{a}}(x;y)$ and $\mathcal{P}_{15}A_{\bar{\nu}}{}^{a\bar{a}}(x;y)$ pairs as well, and the corresponding explicit integrals are found straightforwardly. Suffice it here just to say that

$$\int_{\mathcal{M}} \bar{\varepsilon}^{\bar{\mu}\bar{\nu}\bar{\sigma}}(\mathcal{P}_P A_{\bar{\mu}}{}^{a\bar{a}}(y))\eta_{\bar{a}b}(\mathcal{P}_Q A_{\bar{\nu}}{}^{b\bar{b}}(y))\eta_{\bar{b}c}(\mathcal{P}_3 A_{\bar{\sigma}}{}^{c\bar{c}}(x;y))\eta_{\bar{c}a} \qquad P, Q = 6^*, 15 \qquad (5.1)$$

define the couplings of an effective E_6-singlet four-dimensional Higgs scalar† to $\varphi^{i}(x)$ and $\varphi'^{i}(x)$, the massless fields represented by harmonic forms in $H^1(\mathcal{M}, \mathrm{End}\,\mathcal{T}_{\mathcal{M}})$.

The integral (5.1), non-vanishing in general, is not the one in (4.1) and (4.3), and there is no natural way to associate (5.1) with a (real) contour integral; all of them, however, depend on the background value of $A^{(1)}_{\bar{\nu}}$. If this background value is non-vanishing, the integral in (5.1) acquires a VEV and provides a mass term for $\varphi(x)$ and $\varphi'(x)$. In the basis $\varphi \oplus \varphi'$, the mass matrix takes the form

$$\begin{pmatrix} \mathbf{0} & \mathbf{B} \\ \mathbf{B} & \mathbf{D}_A \end{pmatrix}$$

where the submatrix $\mathbf{B}$ represents $\varphi\varphi'$ mixing terms and $\mathbf{D}_A$ is antisymmetric in the basis of φ'. As long as $\mathbf{B} \neq 0$, the eigenvalues are non-vanishing. This implies that a non-vanishing background configuration of $A^{(1)}_{\bar{\nu}}(x;y)$ provides, in general, masses for all of the 0-modes of $A_{\bar{\nu}}{}^{a\bar{a}}(x;y)$, corresponding to elements of $H^1(\mathcal{M}, \mathrm{End}\,\mathcal{T}_{\mathcal{M}})$. Remarkably, this *qualitatively* resembles the effect of the world-sheet instantons, re-derived by exact string-theory methods [5].

(iv) Just as the standard flux-loop operator, the E_6-singlet contour integral is ineffective on the '27*' represented by the Kähler (1, 1)-form leaving it massless. All the other '27 · 27*' pairs receive superheavy masses, which seems to contradict the exact string-theory result [5] that, barring standard flux-loop effects, all '27 · 27*' pairs remain massless. This, however, was derived for orbifolds, disingularised through blowing up their singular points, so that the resulting space is a Calabi–Yau manifold $\mathcal{M}$. These manifolds are however simply connected, so that there can be no non-vanishing flux-loop integrals. While this reconciles our result with the exact string-theory result, it would certainly be important to check it for the case non-simply-connected Calabi–Yau manifolds. This first of all requires the construction of desingularised orbifolds which are multiply connected and then apply the methods of [5], a task which is clearly beyond the scope of this paper.

† This effective Higgs scalar is different from $\mathcal{H}^{(78)}_{\Gamma}(x)$ but both are generated by an integral over $A^{(1)}_{\bar{\nu}}(x;y)$ on $\mathcal{M}$, with a $\bar{\partial}$-closed form as the kernel.

(v) The 0-modes of the supergravity multiplet do not couple to the fields we have considered here [1] so they do not receive masses by the effects derived here. This also coincides with the exact string-theory results [5].

6. Conclusions

We consider the point-field limit of the $E_8 \times E_8$ heterotic superstring compactified on a generic Calabi–Yau manifold. To complement the existing literature [2], we derive from the original tree-level action the mixed couplings of the (super)fields transforming as 27 and 27* of E_6 to E_6-invariant (super)fields φ and φ', the latter two corresponding to elements of $H^1(\mathcal{M}, \mathrm{End}\, \mathcal{T}_{\mathcal{M}})$. In addition, we derive the couplings of '27 · 27*' pairs to contour integrals over the massive fields $A_{\bar{\nu}}^{(78)}(x;y)$ and $\mathcal{P}_3 A_{\bar{\mu}}{}^{a\bar{a}}(x;y) = A_{\bar{\nu}}^{(1)}(x;y)$.

The coupling to the integral over $A_{\bar{\nu}}^{(78)}(x;y)$ is shown to strikingly resemble, if not equal, the coupling to the (linearised) standard flux-loop operator. The coupling to the integral over $\mathcal{P}_3 A_{\bar{\nu}}{}^{a\bar{a}}(x;y)$ is shown to be analogous to that over $A_{\bar{\nu}}^{(78)}(x;y)$, and both of these define effective four-dimensional Higgs scalars. Suitable non-vanishing background configurations of $A_{\bar{\nu}}^{(78)}(x;y)$ and $\mathcal{P}_3 A_{\bar{\nu}}{}^{a\bar{a}}(x;y)$ are shown to give masses to '27 · 27*' pairs provided $\mathcal{M}$ is multiply connected. A suitably chosen non-vanishing background configuration of $\mathcal{P}_3 A_{\bar{\nu}}{}^{a\bar{a}}(x;y)$ is shown to render all the E_6-singlet scalar (super)fields massive, except for those stemming from the supergravity multiplet, regardless of $\pi_1(\mathcal{M})$.

While the results obtained here *qualitatively* resemble some recent exact string theory results, more *quantitative* results cannot be obtained for a generic Calabi–Yau manifold; hopefully this is possible for concrete examples. The fact that the world-sheet instanton effects, to which the couplings to $\mathcal{P}_3 A_{\bar{\nu}}{}^{a\bar{a}}(x;y)$ seemingly correspond, yield mass terms which are *exponentially* damped with the size of $\mathcal{M}$ (in Planck units) is hard to match in the present analysis. As a matter of fact, this property may be an indication that the world-sheet instanton effects are indeed distinct from those of the integral (5.1). To the best of my understanding however, this can be decided only upon the explicit evaluation of (5.1) which is hopefully possible in a concrete model.

Finally, let me note that the curious property of the couplings to the contour integrals (4.1) and (4.3) (they are orthogonal to the '27*' represented by the Kähler (1, 1)-form of $\mathcal{M}$) does not match the commonly *believed* universality of the flux-loop operator. This mismatch appears to be the only obstruction to interpreting (4.1) and (4.3) as the couplings which generate the couplings to the corresponding flux-loop operators. I do not however know of a proof of such a universality.

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Appendix. The contour integrals

We here address the derivation (and explanation) of the relation (4.2) or rather the more general relation:

$$\int_{\mathcal{M}} (3,2)^{i\bar{i}} \wedge a + \text{CT} \simeq \oint_{\Gamma(i,\bar{i})} A \tag{A1}$$

where the real 1-form A decomposes into a, a holomorphic (0, 1)-form, and a form of conjugate type.

First we study the properties of the RHS of equation (A1). Note first of all that for any 1-cycle Γ on $\mathcal{M}$ and any d-closed 1-form ω_1,

$$\oint_{\Gamma} \omega_1 = \int_{\mathcal{M}} \gamma \wedge \omega_1 \qquad \gamma \in \mathrm{H}^5(\mathcal{M}).$$

However, $\mathrm{H}^5(\mathcal{M}) = \varnothing$ for any Calabi-Yau manifold whereby $\int_\Gamma \omega_1$ must vanish for every closed 1-form ω_1 and 1-cycle Γ on a Calabi-Yau manifold. Therefore, in order for

$$\mathcal{H}_{\Gamma}^{(78)}(x) := \oint_{\Gamma} \mathrm{d}y^m \, A_m^{(78)}(x; y) \neq 0$$

and enable E_6 gauge symmetry breaking, $A_m^{(78)}(x; y)\, \mathrm{d}y^m$ must not be d-closed on $\mathcal{M}$. Therefore, when looking for a background configuration for $A_m^{(78)}(x; y)$ one has to study the space of 1-forms modulo closed 1-forms. Since $\mathrm{H}^1(\mathcal{M})$ vanishes on every Calabi-Yau manifold, a closed 1-form must be an exact one, i.e. one is looking for background gauge fields obeying $\partial^{(y)}_{[m} A^{(78)}_{n]}(x; y) \neq 0$ and therefore equivalent by 'pure gauges': $\partial_m^{(y)} S^{(78)}(x; y)$, for some E_6-algebra-valued scalar $S^{(78)}(x; y)$.

Consider now the first of the integrals in the LHS of equation (A1). The $b_{2,1} \times b_{1,1}$ matrix of (3, 2)-forms consists of $\bar{\partial}$-closed forms since u^i and $v^{\bar{i}}$ are harmonic (and therefore closed as well). Together with the term of conjugate type, the wedge product $u^i \wedge v^{\bar{i}}$ defines a $b_{2,1} \times b_{1,1}$ matrix of d-closed 5-forms† which must be d-exact as $\mathrm{H}^5(\mathcal{M}) = \varnothing$ on any Calabi-Yau manifold. This we write as

$$u^i \wedge v^{\bar{i}} + \text{CT} \simeq \mathrm{d}\vartheta^{i\bar{i}} \qquad \vartheta^{i\bar{i}} \in A_l^4(\mathcal{M}) \tag{A2}$$

where $A_l^r(\mathcal{M})$ is the space of r-forms with coefficients locally integrable on $\mathcal{M}$. One next defines *currents* (see Griffiths and Harris, pp 366-85, in [9] for a rigorous account):

(1) $\mathfrak{F}_\vartheta \in \mathscr{C}^r(\mathcal{M})$ for ϑ an r-form with coefficients locally integrable on $\mathcal{M}$: $\mathfrak{F}_\vartheta(\varphi) := \int_{\mathcal{M}} \vartheta \wedge \varphi$ for $\varphi \in A_c^{(n-r)}(\mathcal{M})$ where $\mathscr{C}^r(\mathcal{M})$ is the space of currents of degree r and $A_c^r(\mathcal{M})$ is the space of $\mathbf{C}^\infty$ r-forms on $\mathcal{M}$;

(2) d: $\mathscr{C}^r(\mathcal{M}) \to \mathscr{C}^{r+1}(\mathcal{M})$: $\mathrm{d}\mathfrak{F}_\vartheta(\varphi) = (-1)^{r+1} \mathfrak{F}_\vartheta(\mathrm{d}\varphi)$ for $\varphi \in A_c^{(n-r-1)}(\mathcal{M})$;

(3) $\mathfrak{F}_\Gamma \in \mathscr{C}^r \mathcal{M}$ for Γ a piecewise smooth $(n-r)$-chain on $\mathcal{M}$: $\mathfrak{F}_\Gamma(\varphi) := \int_\Gamma \varphi$ for $\varphi \in A_c^{(n-r)}(\mathcal{M})$.

It then follows that for any piecewise smooth $(n-r)$-cycle Γ, $\exists \vartheta \in A_l^r(\mathcal{M})$, the restriction of which to $\mathcal{M} - \Gamma$ is $\in A_c^r(\mathcal{M} - \Gamma)$ and for which $\mathrm{d}\vartheta$ is a d-closed $(r+1)$-form on $\mathcal{M} - \Gamma$, such that the equation of currents:

$$\mathfrak{F}_\Gamma = \mathrm{d}\mathfrak{F}_\vartheta - \mathfrak{F}_{\mathrm{d}\vartheta}$$

† This is most easily seen by starting with the fact that the product of a harmonic 3-form and a harmonic 2-form is a d-closed 5-form. Now one decomposes the 3- and the 2-form according to their (p, q) type. Keeping the non-vanishing terms the above statement is confirmed.

holds. In our situation, for every smooth 1-cycle Γ in (A1) there exists a 4-form ϑ such that

$$\oint_\Gamma A^{(78)} = -\int_{\mathcal{M}} \vartheta \wedge dA^{(78)} - \int_{\mathcal{M}} d\vartheta \wedge A^{(78)} \tag{A3}$$

and the identification of this ϑ with the elements of $\vartheta^{i\bar{i}}$ in (A2) is straightforward.

While the second integral in (A3) corresponds to that in the coupling (4.1), the first one is obtained by integration by parts, so that the contour integral appears as a 'surface term'. In this sense, the integral in (4.1) 'generates' that in (4.2). Note that, since $dA \neq 0$, as derived at the beginning of this appendix, (A3) does not imply an *identity* between the Γ-contour integral and the integral with the $(d\vartheta \wedge)$ kernel, but rather an *equivalence* of these, up to integrals with the $(\vartheta \wedge d)$ kernel. Clearly, for a concrete model and an explicit parametrisation of u^i and $v^{\bar{i}}$, this equivalence relation may indeed be found to imply an identity, or else the integrals with the $(\vartheta \wedge d)$ kernel might be estimated or even evaluated.

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